Below is a systems-level mathematical framing for grain sorghum (milo) harvest with a grain platform header (cutter bar + reel + auger + feeder house), carried through rotor/cylinder, concave, cleaning shoe, and tailings. We will use consistent symbols, then build the chain: standing crop → reel contact → cutter bar → platform conveying → feeder house → thresh/separation → cleaning → returns → losses.

0) Core state variables and symbols

0.1 Field and crop variables:

Y= yield, kg grain m^{-2} (or bu ac^{-1} converted) M_g: grain moisture (wet basis fraction)

H = standing crop height, m d_h: head (panicle) characteristic diameter, m m_s: stalk mass per ground area, kg m^{-2} m_h: head + upper stem mass per area, kg m^{-2} $R_{hs} = m_h/m_s$: head-to-stalk ratio entering machine (header-dependent)

Machine kinematics v: ground speed, m s^{-1} W: effective cut width, m Q_{in}: incoming total biomass mass flow, kg s^{-1} Q_g: grain mass flow into the machine, kg s^{-1}

Header Reel radius R_r, angular speed ω_r, tip speed $u_r = \omega_r R_r$ Reel index $\lambda = u_r/v$ (dimensionless) Cutter bar height h_c above ground Auger surface speed u_a

Threshing Rotor/cylinder radius R_c, speed ω_c, tip speed $u_c = \omega_c R_c$ Concave clearance c (characteristic gap) Specific thresh energy E_t per unit mass, J kg^{-1}

Cleaning Fan flow Q_{air}, air velocity field U(x) Grain terminal velocity $V_{t,g}$, chaff terminal velocity $V_{t,c}$ Sieve open area fraction ϕ, sieve length L_s, width W_s

Performance metrics L_H: header loss fraction (grain lost before feeder house) L_T: thresh/separation loss fraction (grain not separated or blown out later due to upstream setting) L_C: cleaning loss fraction (grain carried out the rear) Total loss fraction $L = L_H + L_T + L_C$ (approx; overlap handled later)

0.2 Unified Intake State (Header Abstraction Layer)
A formal definition of the intake condition entering the feeder house, abstracted from header mechanics.

Define:

$$S_{\text{intake}} = \{f_{\text{free}}, f_{\text{attached}}, \phi_{\text{MOG}}, \rho_{\text{mat}}, \sigma_{\text{particle}}\}$$

Where:

- f_{free} = free grain fraction at feeder entry
- f_{attached} = grain still attached to heads
- ϕ_{MOG} = MOG fraction (material other than grain: non-grain mass fraction)
- ρ_{mat} = feeder mat density proxy
- σ_{particle} = effective particle size / aerodynamic variance proxy

Each header defines a mapping:

$H_i(\text{controls, crop state}) \longrightarrow S_{\text{intake}}$

Downstream systems operate on S_{intake}, not directly on header mechanics.

1) Throughput and mass flow fundamentals

1.1 Area rate and biomass inflow

Area harvested per time:

$\dot{A} = vW$

If total above-ground biomass per area is B (kg m^{-2}), then

$Q_{in} = B\,vW$

Grain mass flow (wet basis if you want) is:

$Q_g = Y\,vW$

If you want dry-matter grain flow:

$Q_{g,dm} = \left(1 - M_g\right) Y\,vW$

1.2 Capacity constraints

A simple "shoe/rotor capacity" constraint can be written as

$Q_{in} \leq Q_{\max}(\omega_c, c, Q_{air}, \phi, \dots)$

and in practice, you treat $Q_{\max}$ as an empirical surface that drops when material is wetter, greener, or more stalk-heavy.

2) Milo platform header mathematics

With a dedicated milo header, dominant header loss mechanisms are: Row-guided head capture, head support prior to cut, controlled severing at stem below head, minimal downward pull, and reduced impact energy at intake.

2.1 Row guidance and head capture

Define:

n: number of row units

s_r: row spacing

$W \approx n s_r$: effective cut width

p_{miss}: probability head fails to enter row channel

Capture efficiency:

$\eta_{cap} = 1 - p_{miss}$

Header loss due to miss:

$L_{H,miss} \approx p_{miss}$

In a properly aligned milo header, p_{miss} is typically low and weakly dependent on ground speed compared to grain platform reel mismatch.

2. 2 Head support and controlled cut

Let:

F_c: cutting force applied at stem

h_c: vertical cut location below head

h_{sup}: vertical support point beneath head

M_{bend}: bending moment at head

For a supported cut:

$M_{bend} \approx F_c \cdot (h_c - h_{sup})$

Because $h_c \approx h_{sup}$ in a dedicated milo header (support pan under head before cut):

$M_{bend} \rightarrow$ small

So shatter risk is driven less by bending and more by cut shock:

$p_{sh} = \sigma\big(k\,[F_c - F_0(M_g)]\big)$

where $F_0(M_g)$ increases with moisture.

Expected header shatter:

$L_{H,sh} \approx N_c\, p_{sh}\, f_{seed}$

In practice, $N_c \approx 1$, and p_{sh} is small relative to grain platform and corn header systems.

2. 3 Intake aggressiveness index

u_i: row intake chain speed

Define:

$$\Lambda_{mh} = \frac{u_i}{v}$$

Constraint:

$|\Lambda_{mh} - 1| \leq \epsilon$

Meaning intake should match forward speed to avoid forward drag or rearward pull. Excess mismatch increases shear at stem.

2. 4 Cutter bar height and "head-first" contact fraction

Let the head height above ground be h_h. If cutter bar height h_c is too low relative to head height, more stalk enters, changing R_{hs} and shoe load. If too high, you risk uncut heads or drag.

A simple intake partition model:

$R_{hs} = R_{hs,0}\, g(h_c; h_h)$

where $g(\cdot)$ increases head fraction as you raise h_c (up to practical limits), and $R_{hs,0}$ is a field baseline.

Operationally: raising h_c reduces Q_{in} for the same Q_g, which relaxes downstream constraints.

3) Platform conveying and feeder house: mat density and free grain

Define free grain fraction at feeder entry, f_{free}, as the portion of grain not attached to heads by the time it hits the feeder:

$f_{free} = f_{free,0} + \Delta f_{cut}(F_c, M_g)$

where:

$$\Delta f_{cut} = \sigma\left(c_0 + c_1\left(F_c - F_0(M_g)\right)\right)$$

Because bending is minimized, most free grain arises from brittle heads during cut, not impact.

Thus:

$$Q_{free} = f_{free} Q_g$$

For a milo header:

$f_{free} \ll f_{free,\text{platform}}$ (typically)

Free grain increases cleaning load without needing thresh energy, which can be good or bad depending on shoe capacity and fan setting.

Unlike grain platform or corn header:

- Head-to-stalk ratio R_{hs} is maximized
  Stalk intake is minimized

Q_{in} per unit Q_g is lower

Thus:

$R_{hs} \uparrow$

and intake mass flow still follows your definition:

$$Q_{in} = BvW$$

but effective MOG fraction is reduced.

Mat density proxy:

$$\rho_m \propto \frac{Q_{in}}{u_f W_f}$$

where u_f is feeder chain speed and W_f is feeder width. High ρ_m drives rotor overload and separation inefficiency.

For equal grain yield:

$\rho_{m,\text{milo header}} < \rho_{m,\text{platform}} < \rho_{m,\text{corn}}$

This directly increases rotor margin and separation efficiency.

4) Threshing and separation: energy, probability, and damage

4. 1 Thresh completeness as a saturating function of energy input

Define a specific thresh energy delivered per unit mass entering threshing:

$E_t \approx \alpha\, u_c^2\, \Psi(c)\, \Phi(\rho_m)$

$u_c = \omega_c R_c$ tip speed $\Psi(c)$ increases as clearance tightens (more interaction) $\Phi(\rho_m)$ captures mat density effects (can increase interaction but also cause cushioning and inefficiency)

Then model fraction threshed:

$\eta_{th} = 1 - \exp(-\beta E_t)$

Unthreshed fraction:

$1 - \eta_{th} = \exp(-\beta E_t)$

4.2 Header-to-Rotor Energy Shift

A compact relationship showing how header-induced pre-fracture alters required threshing energy.

Define:

$E_{\text{required}} = E_{\text{base}} \cdot \left(1 - f_{\text{prefracture}}\right)$

Where:

- $f_{\text{prefracture}}$ = effective fraction of grain released or structurally weakened before threshing
    - High for grain platform (reel shatter)
    - Moderate for corn header (bend-induced fracture)
    - Low for milo header (supported cut)

This does not replace the detailed thresh completeness model. It is a scaling bridge.

4. 3 Grain damage (cracking) as a competing risk

Let damage probability increase with rotor tip speed and inverse clearance:

$p_{dam} = \sigma\left(a\,[u_c - u_0] + b\,[c_0 - c] - d\,M_g\right)$

This encodes: higher speed and tighter clearance increase damage, higher moisture reduces brittleness-related cracking risk (within reason).

4. 4 Separation efficiency and residence time

Separation often behaves like a residence-time process. If t_{res} is residence time in separation region:

$$t_{res} \propto \frac{L_{sep}}{u_{ax}}$$

where u_{ax} is axial conveying speed, and L_{sep} is separation length.

A common saturating form:

$$\eta_{sep} = 1 - \exp(-k_s t_{res})$$

Then "separation loss" (grain carried out with MOG) can be approximated as:

$$L_{T,sep} \approx \left(1 - \eta_{sep}\right)(1 - p_{dam})\left(\eta_{th} Q_g\right)$$

(Conceptually: grain that was released but not separated.)

4. 5 Propagation to threshing

Because:

- f_{free} is lower
  R_{hs} is higher
  ρ_m is lower

Thresh energy primarily acts on intact heads:

$$Q_g = Q_{attached} + Q_{free}$$

but here:

$$Q_{attached} \approx Q_g$$

So required E_t may increase slightly compared to grain platform (less pre-fracture).

However, lower mat density improves separation:

$$\Phi(\rho_m) \downarrow \Rightarrow \eta_{sep} \uparrow$$

Net effect:

- Slightly higher thresh demand
  Lower separation loss
  Lower cleaning overload risk

5) Cleaning shoe: air, terminal velocity, and separation capacity

5. 1 Terminal velocity and carryout criterion

A simplified carryout condition compares local air velocity U to grain terminal velocity $V_{t,g}$.

For a particle in turbulent flow, a standard terminal velocity approximation uses drag balance:

$$\frac{1}{2}\rho_{air} C_d A V_t^2 = mg$$

So

$$V_t = \sqrt{\frac{2mg}{\rho_{air} C_d A}}$$

Grain has higher m/A than chaff, so $V_{t,g} > V_{t,c}$.

A crude carryout risk model:

$$p_{carry} = \sigma\left(\gamma \left[U - V_{t,g}\right]\right)$$

Cleaning loss fraction:

$$L_C \approx p_{carry} \frac{Q_g}{Q_{in}} \times \text{(shoe loading factor)}$$

5.2 Pressure drop and fan law scaling

Fan power scales approximately as:

$$P_{fan} \propto Q_{air}\, \Delta p$$

and for similar operating points, fan affinity laws give:

$$Q_{air} \propto n, \quad \Delta p \propto n^2, \quad P_{fan} \propto n^3$$

where n is fan rotational speed. This is why small changes in fan RPM can have outsized effects.

5.3 Sieve capacity as a flux limit

Let J_g be grain flux through sieve area:

$$J_g = \frac{Q_g}{W_s L_s}$$

If J_g exceeds an empirical capacity $J_{g,\max}$ that depends on ϕ, vibration amplitude, and MOG (material other than grain) fraction, you get tailings and/or carryout. A simple overload factor:

$$\Omega = \frac{J_g}{J_{g,\max}(\phi, \text{MOG})}$$

Tailings fraction might rise as:

$$f_{tail} \approx 1 - \exp(-k_t(\Omega - 1)_+)$$

where $(x)_+ = \max(x, 0)$.

6) Tailings recirculation as a damage amplifier

Tailings flow:

$Q_{tail} = f_{tail}\, Q_g$

If tailings are re-threshed, they see additional energy:

$E_{t,eff} = E_t + n_{recirc}\, E_{recirc}$

with n_{recirc} proportional to Q_{tail} and residence dynamics.

Damage risk increases with repeated passes:

$p_{dam,eff} = 1 - (1 - p_{dam})^{(1+n_{recirc})}$

This is a clean way to formalize why "too much tailings" often shows up as cracking, even if initial rotor settings were moderate.

7) Total loss accounting and overlap

A practical decomposition is:

$L = L_H + (1 - L_H)\,[L_T + (1 - L_T)\, L_C]$

This avoids double-counting by applying later losses only to grain that survived earlier stages.

Where: L_H from reel shatter + platform drop + missed heads L_T from unthreshed + separation out back L_C from carryout with chaff

8) Optimization framing: "best settings" as constrained minimization

You can treat setup as minimizing expected loss subject to capacity and grain quality constraints.

Decision vector: $\mathbf{x} = [v,\ u_i,\ h_c,\ \omega_c,\ c,\ Q_{air},\ \phi_{top},\ \phi_{bot}, \ldots]$

Objective (example):

$\min_{\mathbf{x}} \mathbb{E}\big[L\big(\mathbf{x}; M_g, Y, H, \text{lodging}, \ldots\big)\big] + \lambda_{dam}\, \mathbb{E}\big[p_{dam,eff}(\mathbf{x})\big]$

Constraints:

$Q_{in}(\mathbf{x}) \le Q_{\max}(\mathbf{x})$

$\text{sample impurity}(\mathbf{x}) \le I_{\max}$

$p_{carry}(\mathbf{x}) \le p_{carry,\max}$

and practical bounds (machine limits).

The key structural point: for a milo platform, the most impotant upstream control variables are, intake speed ratio Λ_{mh}, cut force F_c, and row alignment (affects p_{miss}).

9) What's actually worth calibrating with your own data

If you ever want this to become predictive rather than conceptual, the minimal calibration set is:

- Header cut-induced shatter curve: $p_{sh}(F_c, M_g)$
- Thresh completeness curve: $\eta_{th}(u_c, c, \rho_m, M_g)$
- capture efficiency curve: $p_{miss}(v, \delta_{row})$
- Carryout curve: $p_{carry}(U, M_g)$ plus a proxy linking fan RPM to effective shoe velocity
- U Tailings feedback: $f_{tail}(\Omega)$ and a
- Cracking increase vs Q_{tail}

With even rough field measurements (pan counts and tank sample), you can fit these as low-parameter logistic or saturating exponentials.