Below is a systems-level mathematical framing for grain sorghum (milo) harvest with a grain platform header (cutter bar + reel + auger + feeder house), carried through rotor/cylinder, concave, cleaning shoe, and tailings. We will use consistent symbols, then build the chain: standing crop → reel contact → cutter bar → platform conveying → feeder house → thresh/separation → cleaning → returns → losses.

0) Core state variables and symbols

0.1 Field and crop variables:

Y= yield, kg grain m^{-2} (or bu ac^{-1} converted) M_g: grain moisture (wet basis fraction)

H = standing crop height, m d_h: head (panicle) characteristic diameter, m m_s: stalk mass per ground area, kg m^{-2} m_h: head + upper stem mass per area, kg m^{-2} $R_{hs} = m_h/m_s$: head-to-stalk ratio entering machine (header-dependent)

Machine kinematics v: ground speed, m s^{-1} W: effective cut width, m Q_{in}: incoming total biomass mass flow, kg s^{-1} Q_g: grain mass flow into the machine, kg s^{-1}

Header Reel radius R_r, angular speed ω_r, tip speed $u_r = \omega_r R_r$ Reel index $\lambda = u_r/v$ (dimensionless) Cutter bar height h_c above ground Auger surface speed u_a

Threshing Rotor/cylinder radius R_c, speed ω_c, tip speed $u_c = \omega_c R_c$ Concave clearance c (characteristic gap) Specific thresh energy E_t per unit mass, J kg^{-1}

Cleaning Fan flow Q_{air}, air velocity field U(x) Grain terminal velocity $V_{t,g}$, chaff terminal velocity $V_{t,c}$ Sieve open area fraction ϕ, sieve length L_s, width W_s

Performance metrics L_H: header loss fraction (grain lost before feeder house) L_T: thresh/separation loss fraction (grain not separated or blown out later due to upstream setting) L_C: cleaning loss fraction (grain carried out the rear) Total loss fraction $L = L_H + L_T + L_C$ (approx; overlap handled later)

0.2 Unified Intake State (Header Abstraction Layer)
A formal definition of the intake condition entering the feeder house, abstracted from header mechanics.

Define:

$$S_{\text{intake}} = \{f_{\text{free}}, f_{\text{attached}}, \phi_{\text{MOG}}, \rho_{\text{mat}}, \sigma_{\text{particle}}\}$$

Where:

- f_{free} = free grain fraction at feeder entry
- f_{attached} = grain still attached to heads
- ϕ_{MOG} = MOG fraction (material other than grain: non-grain mass fraction)
- ρ_{mat} = feeder mat density proxy
- σ_{particle} = effective particle size / aerodynamic variance proxy

Each header defines a mapping:

$H_i(\text{controls, crop state}) \longrightarrow S_{\text{intake}}$

Downstream systems operate on S_{intake}, not directly on header mechanics.

1) Throughput and mass flow fundamentals

1.1 Area rate and biomass inflow

Area harvested per time:

$\dot{A} = vW$

If total above-ground biomass per area is B (kg m^{-2}), then

$Q_{in} = B\,vW$

Grain mass flow (wet basis if you want) is:

$Q_g = Y\,vW$

If you want dry-matter grain flow:

$Q_{g,dm} = \left(1 - M_g\right) Y\,vW$

1.2 Capacity constraints

A simple "shoe/rotor capacity" constraint can be written as

$Q_{in} \le Q_{\max}(\omega_c, c, Q_{air}, \phi, \dots)$

and in practice, you treat $Q_{\max}$ as an empirical surface that drops when material is wetter, greener, or more stalk-heavy.

2) Grain platform header mathematics

With a grain platform, the dominant header-specific loss driver in milo is often reel-induced shatter and impact at the head, plus platform drop (heads or loose grain falling forward or under the platform).

2.1 Reel kinematics: reel index and relative impact speed

Reel tip speed:

$u_r = \omega_r R_r$

Reel index:

$\lambda = \dfrac{u_r}{v}$

A first-order relative velocity at first contact (head vs tine) can be approximated as:

$v_{rel} \approx |u_r - v|$

(That is crude but useful for scaling shatter risk with λ.)

If you model the tine's approach angle θ at contact, then:

$v_{rel,n} \approx |u_r \cos\theta - v|$

where $v_{rel,n}$ is the normal component that drives impulsive loading (and shatter).

2.2 Shatter as a brittle-fracture probability

A practical model: shatter fraction per head contact is a function of impact impulse or stress exceeding a moisture-dependent threshold.

Let I be impulse per contact:

$I \approx m_{eff}\, v_{rel,n}$

where m_{eff} is an effective interacting mass (tine + local head segment).

Assume a threshold impulse $I_0(M_g)$ that increases with moisture. One smooth risk model:

$p_{sh} = \sigma\big(k\,[I - I_0(M_g)]\big)$

where $\sigma(z) = \frac{1}{1+e^{-z}}$ is logistic, and k controls steepness.

Expected header shatter loss fraction from reel contact:

$L_{H,sh} \approx N_c\, p_{sh}\, f_{seed}$

N_c: expected number of meaningful contacts per head (depends on reel geometry and lodging) f_{seed}: fraction of grain exposed to shatter per contact event (calibrated)

2.3 Reel-to-ground-speed tuning target

Rather than "a best λ," define a constraint:

$v_{rel,n} \le v_{crit}(M_g)$

or equivalently,

$|\omega_r R_r \cos\theta - v| \le v_{crit}(M_g)$

This captures the idea that drier heads demand lower impact.

2.4 Cutter bar height and "head-first" contact fraction

Let the head height above ground be h_h. If cutter bar height h_c is too low relative to head height, more stalk enters, changing R_{hs} and shoe load. If too high, you risk uncut heads or drag.

A simple intake partition model:

$$R_{hs} = R_{hs,0}\, g(h_c; h_h)$$

where $g(\cdot)$ increases head fraction as you raise h_c (up to practical limits), and $R_{hs,0}$ is a field baseline.

Operationally: raising h_c reduces Q_{in} for the same Q_g, which relaxes downstream constraints.

3) Platform conveying and feeder house: mat density and free grain

Define free grain fraction at feeder entry, f_{free}, as the portion of grain not attached to heads by the time it hits the feeder:

$$f_{free} = f_{free,0} + \Delta f_{reel}(\lambda, M_g) + \Delta f_{drop}(\text{platform})$$

This matters because free grain behaves differently in threshing and cleaning.

A useful load variable for the shoe is:

$$Q_{free} = f_{free}\, Q_g$$

Free grain increases cleaning load without needing thresh energy, which can be good or bad depending on shoe capacity and fan setting.

Mat mass per unit area on the feeder chain (a proxy for choke risk):

$$\rho_m \propto \frac{Q_{in}}{u_f W_f}$$

where u_f is feeder chain speed and W_f is feeder width. High ρ_m drives rotor overload and separation inefficiency.

4) Threshing and separation: energy, probability, and damage

4. 1 Thresh completeness as a saturating function of energy input

Define a specific thresh energy delivered per unit mass entering threshing:

$$E_t \approx \alpha\, u_c^2\, \Psi(c)\, \Phi(\rho_m)$$

$u_c = \omega_c R_c$ tip speed $\Psi(c)$ increases as clearance tightens (more interaction) $\Phi(\rho_m)$ captures mat density effects (can increase interaction but also cause cushioning and inefficiency)

Then model fraction threshed:

$$\eta_{th} = 1 - \exp(-\beta E_t)$$

Unthreshed fraction:

$$1 - \eta_{th} = \exp(-\beta E_t)$$

4.2 Header-to-Rotor Energy Shift

A compact relationship showing how header-induced pre-fracture alters required threshing energy.

Define:

$$E_{\text{required}} = E_{\text{base}} \cdot \left(1 - f_{\text{prefracture}}\right)$$

Where:

- $f_{\text{prefracture}}$ = effective fraction of grain released or structurally weakened before threshing
    - High for grain platform (reel shatter)
    - Moderate for corn header (bend-induced fracture)
    - Low for milo header (supported cut)

This does not replace the detailed thresh completeness model. It is a scaling bridge.

4. 3 Grain damage (cracking) as a competing risk

Let damage probability increase with rotor tip speed and inverse clearance:

$$p_{dam} = \sigma\left(a\,[u_c - u_0] + b\,[c_0 - c] - d\,M_g\right)$$

This encodes: higher speed and tighter clearance increase damage, higher moisture reduces brittleness-related cracking risk (within reason).

4. 4 Separation efficiency and residence time

Separation often behaves like a residence-time process. If t_{res} is residence time in separation region:

$$t_{res} \propto \frac{L_{sep}}{u_{ax}}$$

where u_{ax} is axial conveying speed, and L_{sep} is separation length.

A common saturating form:

$$\eta_{sep} = 1 - \exp(-k_s t_{res})$$

Then "separation loss" (grain carried out with MOG) can be approximated as:

$$L_{T,sep} \approx \left(1 - \eta_{sep}\right)\left(1 - p_{dam}\right)\left(\eta_{th} Q_g\right)$$

(Conceptually: grain that was released but not separated.)

4. 5 How header choice propagates here (grain platform case)

Grain platform tends to raise f_{free} via reel shatter. That shifts the required E_t downward for the same Q_g (less attachment to break), but increases shoe load and the risk that free grain gets carried out if fan is wrong.

One useful split:

$$Q_g = Q_{attached} + Q_{free}$$

with

$$Q_{free} = f_{free} Q_g, \quad Q_{attached} = (1 - f_{free}) Q_g$$

Then thresh energy only needs to act strongly on $Q_{attached}$, while cleaning must handle Q_{free} plus released grain.

5) Cleaning shoe: air, terminal velocity, and separation capacity

5. 1 Terminal velocity and carryout criterion

A simplified carryout condition compares local air velocity U to grain terminal velocity $V_{t,g}$.

For a particle in turbulent flow, a standard terminal velocity approximation uses drag balance:

$$\frac{1}{2} \rho_{air} C_d A V_t^2 = mg$$

So

$$V_t = \sqrt{\frac{2mg}{\rho_{air} C_d A}}$$

Grain has higher m/A than chaff, so $V_{t,g} > V_{t,c}$.

A crude carryout risk model:

$$p_{carry} = \sigma(\gamma [U - V_{t,g}])$$

Cleaning loss fraction:

$$L_C \approx p_{carry} \frac{Q_g}{Q_{in}} \times \text{(shoe loading factor)}$$

5. 2 Pressure drop and fan law scaling

Fan power scales approximately as:

$$P_{fan} \propto Q_{air} \Delta p$$

and for similar operating points, fan affinity laws give:

$$Q_{air} \propto n, \quad \Delta p \propto n^2, \quad P_{fan} \propto n^3$$

where n is fan rotational speed. This is why small changes in fan RPM can have outsized effects.

5. 3 Sieve capacity as a flux limit

Let J_g be grain flux through sieve area:

$$J_g = \frac{Q_g}{W_s L_s}$$

If J_g exceeds an empirical capacity $J_{g,\max}$ that depends on ϕ, vibration amplitude, and MOG (material other than grain) fraction, you get tailings and/or carryout. A simple overload factor:

$$\Omega = \frac{J_g}{J_{g,\max}(\phi, \mathrm{MOG})}$$

Tailings fraction might rise as:

$$f_{tail} \approx 1 - \exp(-k_t(\Omega - 1)_+)$$

where $(x)_+ = \max(x, 0)$.

5. 4 How grain platform intake changes shoe tuning

Because grain platform can increase Q_{free} (loose grain), the front-loaded grain flux into the shoe can increase even if total yield is constant. That pushes J_g up and raises tailings risk unless fan, sieve opening, or throughput are adjusted.

6) Tailings recirculation as a damage amplifier

Tailings flow:

$$Q_{tail} = f_{tail}\, Q_g$$

If tailings are re-threshed, they see additional energy:

$$E_{t,eff} = E_t + n_{recirc}\, E_{recirc}$$

with n_{recirc} proportional to Q_{tail} and residence dynamics.

Damage risk increases with repeated passes:

$$p_{dam,eff} = 1 - (1 - p_{dam})^{(1+n_{recirc})}$$

This is a clean way to formalize why "too much tailings" often shows up as cracking, even if initial rotor settings were moderate.

7) Total loss accounting and overlap

A practical decomposition is:

$$L = L_H + (1 - L_H)\,[L_T + (1 - L_T)\,L_C]$$

This avoids double-counting by applying later losses only to grain that survived earlier stages.

Where: L_H from reel shatter + platform drop + missed heads L_T from unthreshed + separation out back L_C from carryout with chaff

8) Optimization framing: "best settings" as constrained minimization

You can treat setup as minimizing expected loss subject to capacity and grain quality constraints.

Decision vector: $\mathbf{x} = [v,\ \omega_r,\ h_c,\ \omega_c,\ c,\ Q_{air}$ (or fan RPM), $\phi_{top},\ \phi_{bot},\ \ldots]$

Objective (example):

$$\min_{\mathbf{x}} \mathbb{E}\big[L\big(\mathbf{x}; M_g, Y, H,\text{lodging}, \ldots\big)\big] + \lambda_{dam}\,\mathbb{E}\big[p_{dam,eff}(\mathbf{x})\big]$$

Constraints:

$$Q_{in}(\mathbf{x}) \le Q_{\max}(\mathbf{x})$$

$$\text{sample impurity}(\mathbf{x}) \le I_{\max}$$

$$p_{carry}(\mathbf{x}) \le p_{carry,\max}$$

and practical bounds (machine limits).

The key structural point: for a grain platform, ω_r and v (through λ) sit upstream and shift the feasible region downstream, especially through f_{free} and mat density ρ_m.

9) What's actually worth calibrating with your own data

If you ever want this to become predictive rather than conceptual, the minimal calibration set is: Header shatter curve: $p_{sh}(\lambda, M_g)$ or directly $L_{H,sh}(\lambda, M_g)$ Thresh completeness curve: $\eta_{th}(u_c, c, \rho_m, M_g)$ Carryout curve: $p_{carry}(U, M_g)$ plus a proxy linking fan RPM to effective shoe velocity U Tailings feedback: $f_{tail}(\Omega)$ and a cracking increase vs Q_{tail}

With even rough field measurements (pan counts and tank sample), you can fit these as low-parameter logistic or saturating exponentials.