Below is a systems-level mathematical framing for grain sorghum (milo) harvest with a grain platform header (cutter bar + reel + auger + feeder house), carried through rotor/cylinder, concave, cleaning shoe, and tailings. We will use consistent symbols, then build the chain: standing crop → reel contact → cutter bar → platform conveying → feeder house → thresh/separation → cleaning → returns → losses.

0) Core state variables and symbols

0.1 Field and crop variables:

Y= yield, kg grain m^{-2} (or bu ac^{-1} converted) M_g: grain moisture (wet basis fraction)

H = standing crop height, m d_h: head (panicle) characteristic diameter, m m_s: stalk mass per ground area, kg m^{-2} m_h: head + upper stem mass per area, kg m^{-2} $R_{hs} = m_h/m_s$: head-to-stalk ratio entering machine (header-dependent)

Machine kinematics v: ground speed, m s^{-1} W: effective cut width, m Q_{in}: incoming total biomass mass flow, kg s^{-1} Q_g: grain mass flow into the machine, kg s^{-1}

Header Reel radius R_r, angular speed ω_r, tip speed $u_r = \omega_r R_r$ Reel index $\lambda = u_r/v$ (dimensionless) Cutter bar height h_c above ground Auger surface speed u_a

Threshing Rotor/cylinder radius R_c, speed ω_c, tip speed $u_c = \omega_c R_c$ Concave clearance c (characteristic gap) Specific thresh energy E_t per unit mass, J kg^{-1}

Cleaning Fan flow Q_{air}, air velocity field U(x) Grain terminal velocity $V_{t,g}$, chaff terminal velocity $V_{t,c}$ Sieve open area fraction ϕ, sieve length L_s, width W_s

Performance metrics L_H: header loss fraction (grain lost before feeder house) L_T: thresh/separation loss fraction (grain not separated or blown out later due to upstream setting) L_C: cleaning loss fraction (grain carried out the rear) Total loss fraction $L = L_H + L_T + L_C$ (approx; overlap handled later)

n: number of row units engaged (dimensionless)

s_r: row spacing, m

$W \approx n s_r$: effective header width (replace platform W if you were using full platform width)

u_{ch}: gathering chain speed, m s^{-1}

u_{sr}: snapping roll surface speed, m s^{-1}

Δu_{sr}: differential roll speed if used, m s^{-1}

g_{sr}: roll gap (effective), m

F_{sr}: effective downward pull force applied to stalk by rolls, N

τ_{sr}: roll torque (per row unit), N m

σ_s: stalk tensile strength, Pa

A_s: stalk cross-sectional area, m^2

h_{snap}: vertical location of "snapping zone" relative to head, m

M_{bend}: bending moment at head attachment region, N m

p_{miss}: probability head/stalk misses row unit capture (row guidance issue)

0.2 Unified Intake State (Header Abstraction Layer)
A formal definition of the intake condition entering the feeder house, abstracted from header mechanics.

Define:

$$S_{\text{intake}} = \{f_{\text{free}}, f_{\text{attached}}, \phi_{\text{MOG}}, \rho_{\text{mat}}, \sigma_{\text{particle}}\}$$

Where:

- f_{free} = free grain fraction at feeder entry
- f_{attached} = grain still attached to heads
- ϕ_{MOG} = MOG fraction (material other than grain: non-grain mass fraction)
- ρ_{mat} = feeder mat density proxy
- σ_{particle} = effective particle size / aerodynamic variance proxy

Each header defines a mapping:

$$H_i(\text{controls},\ \text{crop state}) \longrightarrow S_{\text{intake}}$$

Downstream systems operate on S_{intake}, not directly on header mechanics.

1) Throughput and mass flow fundamentals

1. 1 Area rate and biomass inflow

Area harvested per time:

$$\dot{A} = vW$$

If total above-ground biomass per area is B (kg m^{-2}), then

$$Q_{in} = B\, vW$$

Grain mass flow (wet basis if you want) is:

$$Q_g = Y\, vW$$

If you want dry-matter grain flow:

$$Q_{g,dm} = \left(1 - M_g\right) Y\, vW$$

1. 2 Capacity constraints

A simple "shoe/rotor capacity" constraint can be written as

$$Q_{in} \le Q_{\max}(\omega_c, c, Q_{air}, \phi, \dots)$$

and in practice, you treat $Q_{\max}$ as an empirical surface that drops when material is wetter, greener, or more stalk-heavy.

2) Grain platform header mathematics

With a corn header, the dominant loss mechanisms in milo are: row-unit capture/miss, downward stalk pull vs head support mismatch, and bending/tearing at the panicle zone causing shatter.

2.1 Row capture and alignment (replaces reel kinematics)

A simple alignment/capture efficiency term:

$$\eta_{cap} = 1 - p_{miss}$$

and effective harvested width becomes:

$$W \approx \eta_{cap}\, n\, s_r$$

If you want to make p_{miss} depend on speed and row guidance tolerance, a workable form is:

$$p_{miss} = \sigma(a_0 + a_1(v - v_0) + a_2\, \delta_{row})$$

where δ_{row} is a lateral misalignment metric (row wandering, crop leaning, operator guidance error), and $\sigma(\cdot)$ is logistic as in your document.

2.2 Downward pull intensity (new governing "header index")

Define a dimensionless pull-to-strength group:

$$\Pi_{pull} = \frac{F_{sr}}{\sigma_s A_s}$$

Interpretation: when Π_{pull} rises above ~ 1 (conceptually), stalk failure becomes likely somewhere in the pull path. In milo, that "failure" often manifests as head-zone stress and shatter rather than a clean snap at a designed ear zone.

Relate pull force to roll kinematics and geometry:
$F_{sr} \approx \kappa_F \frac{\tau_{sr}}{r_{sr}}$ and $\tau_{sr} \propto T_{drive}\, f(g_{sr}, \text{surface}, \text{moisture})$

You rarely know τ_{sr} directly, but this provides a scaling handle: roll aggressiveness is effectively controlled by roll speed, gap, surface, and crop condition.

A practical alternative to force is a kinematic aggressiveness index:
$\Lambda_{sr} = \frac{u_{sr}}{v}$

This is analogous in spirit to the reel index λ, but it governs downward pull rate relative to forward feed, not impact speed.

2.3 Shatter as bending/tear probability (replaces reel-impact shatter model)

Instead of impulse at a reel tine, you have bending moment and shear at the head region driven by unsupported head above the snapping zone.

A simple bending moment proxy:
$M_{bend} \approx F_{sr}\, h_{snap}$

where h_{snap} increases when the "effective snap point" sits farther below the head (i.e., more unsupported lever arm).

Let a moisture-dependent threshold moment $M_0(M_g)$ increase with moisture. Then:
$p_{sh} = \sigma\big(k\,[M_{bend} - M_0(M_g)]\big)$

Expected header shatter loss fraction (same structure as your reel model, different driver):
$L_{H,sh} \approx N_{int}\, p_{sh}\, f_{seed}$

N_{int}: number of meaningful head-zone stress events per stalk entering the rolls (often near 1, but increases with lodging and recapture events)

2.4 Loss representation for corn header (replaces "platform drop" logic)

Total header loss now decomposes as:
$L_H = L_{H,miss} + \big(1 - L_{H,miss}\big)\big[L_{H,sh} + L_{H,drop}\big]$

where:
$L_{H,miss} \approx p_{miss}$ (missed capture / row guidance)

$L_{H,sh}$ from bending-moment model above

$L_{H,drop}$ covers heads/seed that detach and fall below the deck during pull (often coupled to Π_{pull} and M_g)

A simple coupled form for drop loss is:
$L_{H,drop} = \sigma\big(b_0 + b_1(\Pi_{pull} - 1) + b_2(v - v_0) - b_3 M_g\big)$

2.5 "Tuning targets" for a corn header (replaces reel-to-ground constraint)

Instead of constraining $v_{rel,n}$, you constrain a pull/bend driver:
$M_{bend} \le M_{crit}(M_g)$ or $\Pi_{pull} \le \Pi_{crit}(M_g)$

Operational meaning: as heads get drier and more brittle, you need to keep the snap/pull mechanics from creating excessive head-zone bending.

3) Platform conveying and feeder house: mat density and free grain

Define free grain fraction at feeder entry, f_{free}, as the portion of grain not attached to heads by the time it hits the feeder:

$f_{free} = f_{free,0} + \Delta f_{sr}(\Pi_{pull}, M_g, h_{snap}) + \Delta f_{miss}(p_{miss})$

One workable smooth model:

$$\Delta f_{sr} = \sigma\left(c_0 + c_1(\Pi_{pull} - 1) + c_2 \frac{h_{snap}}{H} - c_3 M_g\right)$$

and
$\Delta f_{miss} \approx \xi\, p_{miss}$

(where ξ reflects how much "miss" turns into loose grain vs intact heads left standing).
Then the shoe load variable remains the same definition:
$Q_{free} = f_{free} Q_g$

Mat density proxy stays structurally identical:

$$\rho_m \propto \frac{Q_{in}}{u_f W_f}$$

but note the upstream effect: corn headers typically shift Q_{in} upward for the same Q_g via a lower R_{hs} (more stalk intake), so ρ_m tends to rise even before you touch rotor settings.

4) Threshing and separation: energy, probability, and damage

4.1 Thresh completeness as a saturating function of energy input

Define a specific thresh energy delivered per unit mass entering threshing:

$E_t \approx \alpha\, u_c^2\, \Psi(c)\, \Phi(\rho_m)$

$u_c = \omega_c R_c$ tip speed $\Psi(c)$ increases as clearance tightens (more interaction) $\Phi(\rho_m)$ captures mat density effects (can increase interaction but also cause cushioning and inefficiency)

Then model fraction threshed:

$\eta_{th} = 1 - \exp(-\beta E_t)$

Unthreshed fraction:

$1 - \eta_{th} = \exp(-\beta E_t)$

4.2 Header-to-Rotor Energy Shift

A compact relationship showing how header-induced pre-fracture alters required threshing energy.

Define:

$$E_{\text{required}} = E_{\text{base}} \cdot \left(1 - f_{\text{prefracture}}\right)$$

Where:

- $f_{\text{prefracture}}$ = effective fraction of grain released or structurally weakened before threshing
    - High for grain platform (reel shatter)
    - Moderate for corn header (bend-induced fracture)
    - Low for milo header (supported cut)

This does not replace the detailed thresh completeness model. It is a scaling bridge.

4.3 Grain damage (cracking) as a competing risk

Let damage probability increase with rotor tip speed and inverse clearance:

$p_{dam} = \sigma\big(a\,[u_c - u_0] + b\,[c_0 - c] - d\,M_g\big)$

This encodes: higher speed and tighter clearance increase damage, higher moisture reduces brittleness-related cracking risk (within reason).

4.4 Separation efficiency and residence time

Separation often behaves like a residence-time process. If t_{res} is residence time in separation region:

$t_{res} \propto \frac{L_{sep}}{u_{ax}}$

where u_{ax} is axial conveying speed, and L_{sep} is separation length.

A common saturating form:

$\eta_{sep} = 1 - \exp(-k_s t_{res})$

Then "separation loss" (grain carried out with MOG) can be approximated as:

$L_{T,sep} \approx \left(1 - \eta_{sep}\right)\left(1 - p_{dam}\right)\left(\eta_{th} Q_g\right)$

(Conceptually: grain that was released but not separated.)

4. 5 How header choice propagates here (corn header)

Corn header tends to alter intake in three coupled ways:

Higher MOG per unit grain (stalk-heavy intake): increases Q_{in} at fixed Q_g, raising ρ_m and reducing effective separation margin.

Mechanically induced free grain via pull/bend mismatch: increases f_{free} through $\Delta f_{sr}\left(\Pi_{pull}, M_g, h_{snap}\right)$.

Wider particle-size distribution (fragments, partial heads): increases overlap between "grain-like" and "chaff-like" aerodynamic behavior in cleaning.

The attached/free split still applies:
$Q_g = Q_{attached} + Q_{free}, \quad Q_{free} = f_{free} Q_g, \quad Q_{attached} = \left(1 - f_{free}\right) Q_g$

But the consequence differs:

Because Q_{in} rises (stalk load), shoe capacity and separation residence time become more limiting even if Q_{free} is high.

Because fragments increase, cleaning carryout risk can rise at the same fan setting (effective V_t distribution broadens), even though the underlying terminal-velocity math is unchanged.

5) Cleaning shoe: air, terminal velocity, and separation capacity

5. 1 Terminal velocity and carryout criterion

A simplified carryout condition compares local air velocity U to grain terminal velocity $V_{t,g}$.

For a particle in turbulent flow, a standard terminal velocity approximation uses drag balance:

$\frac{1}{2} \rho_{air} C_d A V_t^2 = mg$

So

$$V_t = \sqrt{\frac{2mg}{\rho_{air} C_d A}}$$

Grain has higher m/A than chaff, so $V_{t,g} > V_{t,c}$.

A crude carryout risk model:

$$p_{carry} = \sigma\left(\gamma \left[U - V_{t,g}\right]\right)$$

Cleaning loss fraction:

$$L_C \approx p_{carry} \frac{Q_g}{Q_{in}} \times \text{(shoe loading factor)}$$

5.2 Pressure drop and fan law scaling

Fan power scales approximately as:

$$P_{fan} \propto Q_{air}\, \Delta p$$

and for similar operating points, fan affinity laws give:

$$Q_{air} \propto n, \quad \Delta p \propto n^2, \quad P_{fan} \propto n^3$$

where n is fan rotational speed. This is why small changes in fan RPM can have outsized effects.

5.3 Sieve capacity as a flux limit

Let J_g be grain flux through sieve area:

$$J_g = \frac{Q_g}{W_s L_s}$$

If J_g exceeds an empirical capacity $J_{g,\max}$ that depends on ϕ, vibration amplitude, and MOG (material other than grain) fraction, you get tailings and/or carryout. A simple overload factor:

$$\Omega = \frac{J_g}{J_{g,\max}(\phi, \text{MOG})}$$

Tailings fraction might rise as:

$$f_{tail} \approx 1 - \exp(-k_t(\Omega - 1)_+)$$

where $(x)_+ = \max(x, 0)$.

5.4 How grain platform intake changes shoe tuning

Because grain platform can increase Q_{free} (loose grain), the front-loaded grain flux into the shoe can increase even if total yield is constant. That pushes J_g up and raises tailings risk unless fan, sieve opening, or throughput are adjusted.

6) Tailings recirculation as a damage amplifier

Tailings flow:

$$Q_{tail} = f_{tail}\, Q_g$$

If tailings are re-threshed, they see additional energy:

$$E_{t,eff} = E_t + n_{recirc}\, E_{recirc}$$

with n_{recirc} proportional to Q_{tail} and residence dynamics.

Damage risk increases with repeated passes:

$$p_{dam,eff} = 1 - (1 - p_{dam})^{(1+n_{recirc})}$$

This is a clean way to formalize why "too much tailings" often shows up as cracking, even if initial rotor settings were moderate.

7) Total loss accounting and overlap

A practical decomposition is:

$$L = L_H + (1 - L_H)\,[L_T + (1 - L_T)\, L_C]$$

This avoids double-counting by applying later losses only to grain that survived earlier stages.

Where: L_H from reel shatter + platform drop + missed heads L_T from unthreshed + separation out back L_C from carryout with chaff

8) Optimization framing: "best settings" as constrained minimization

You can treat setup as minimizing expected loss subject to capacity and grain quality constraints.

Decision vector: $\mathbf{x} = [v,\ u_{sr},\ g_{sr},\ (\Delta u_{sr}),\ \omega_c,\ c,\ Q_{air},\ \phi_{top},\ \phi_{bot}, \ldots]$

Objective (example):

$$\min_{\mathbf{x}} \mathbb{E}\big[L(\mathbf{x}; M_g, Y, H, \text{lodging}, \ldots)\big] + \lambda_{dam}\, \mathbb{E}\big[p_{dam,eff}(\mathbf{x})\big]$$

Constraints:

$$Q_{in}(\mathbf{x}) \le Q_{\max}(\mathbf{x})$$

$$\text{sample impurity}(\mathbf{x}) \le I_{\max}$$

$$p_{carry}(\mathbf{x}) \le p_{carry,\max}$$

and practical bounds (machine limits).

Replace the “key structural point” (reel index drives feasible region) with:

The key structural point: for a corn header, pull/bend intensity and row capture sit upstream and shift the feasible region downstream primarily through Q_{in}, ρ_m, and f_{free}, with dominant control variables Π_{pull}, $\Lambda_{sr} = u_{sr}/v$, and p_{miss}.

9) **What’s actually worth calibrating with your own data**

If you ever want this to become predictive rather than conceptual, the minimal calibration set is:

Corn-header shatter / free-grain curve: $p_{sh}(M_{bend}, M_g)$ or $L_{H,sh}(\Pi_{pull}, M_g, h_{snap})$

Row capture curve $p_{miss}(v, \delta_{row})$

Thresh completeness curve: $\eta_{th}(u_c, c, \rho_m, M_g)$

Carryout curve: $p_{carry}(U, M_g)$ plus a proxy linking fan RPM to effective shoe velocity

U Tailings feedback: $f_{tail}(\Omega)$

Cracking increase vs Q_{tail}

With even rough field measurements (pan counts and tank sample), you can fit these as low-parameter logistic or saturating exponentials.